ASOL MATHEMATICS SCOPE AND SEQUENCE MATRIX: GRADE 3

MATHEMATICS ASOL SUMMARY MATRIX
Based on the 2009 *Mathematics* Standards of Learning

Reporting Category	Grade 3	Grade 4	Grade 5	Grade 6	Grade 7	Grade 8	High School
Number, Number Sense, Computation and Estimation	3M-NSCE 1 3M-NSCE 2 3M-NSCE 3 3M-NSCE 4 3M-NSCE 5 3M-NSCE 6 3M-NSCE 7	4M-NSCE 1 4M-NSCE 2 4M-NSCE 3 4M-NSCE 4 4M-NSCE 5	5M-NSCE 1 5M-NSCE 2 5M-NSCE 3 5M-NSCE 4	6M-NSCE 1 6M-NSCE 2 6M-NSCE 3 6M-NSCE 4 6M-NSCE 5	7M-NSCE 1 7M-NSCE 2 7M-NSCE 3	8M-NSCE 1 8M-NSCE 2 8M-NSCE 3	
Measurement and Geometry	3M-MG 1 3M-MG 2 3M-MG 3 3M-MG 4	4M-MG 1 4M-MG 2 4M-MG 3	5M-MG 1	6M-MG 1	7M-MG 1 7M-MG 2	8M-MG 1 8M-MG 2 8M-MG 3	
Probability, Statistics, Patterns, Functions, and Algebra	3M-PSPFA 1 3M-PSPFA 2 3M-PSPFA 3	4M-PSPFA 1	5M-PSPFA 1 5M-PSPFA 2	6M-PSPFA 1 6M-PSPFA 2 6M-PSPFA 3	7M-PSPFA 1 7M-PSPFA 2 7M-PSPFA 3	8M-PSPFA 1 8M-PSPFA 2 8M-PSPFA 3 8M-PSPFA 4	
Expressions and Operations							HSM-EO 1 HSM-EO 2
Equations and Inequalities							HSM-EI 1 HSM-EI 2 HSM-EI 3
Functions and Statistics							HSM-FS 1 HSM-FS 2 HSM-FS 3 HSM-FS 4

REPORTING CATEGORIES	GRADE 3 ASOL BLUEPRINT	UNDERSTANDING THE STANDARD
	3M-NSCE 1 (SOL 3.1)	• The structure of the Base-10 number system is based upon a simple pattern of tens, where each place is ten times the value of the place to its right. This is known as a ten-to-one place value relationship. • The structure of the Base-10 blocks is based on the ten-to-one place value relationship (e.g., 10 units make a long, 10 longs make a flat, 10 flats make a cube). • Place value refers to the value of each digit and depends upon the position of the digit in the number. In the number 7,864, the eight is in the hundreds place, and the value of the 8 is eight hundred. • Flexibility in thinking about numbers — or "decomposition" of numbers (e.g., 12,345 is 123 hundreds, 4 tens, and 5 ones) — is critical and supports understandings essential to multiplication and division. • Whole numbers may be written in a variety of formats: – Standard: 123,456; – Written: one hundred twenty-three thousand, four hundred fifty-six; and – Expanded: $(1 \times 100{,}000) + (2 \times 10{,}000) + (3 \times 1{,}000) + (4 \times 100) + (5 \times 10) + (6 \times 1)$. • Numbers are arranged into groups of three places called *periods* (ones, thousands, millions, and so on). Places within the periods repeat (hundreds, tens, ones). Commas are used to separate the periods. Knowing the place value and period of a number helps students find the value of a digit in any number as well as read and write numbers. • To read a whole number through the hundred thousands place, – read the digits to the first comma; – say the name of the period (e.g., "thousands"); then – read the last three digits, but do not say the name of the ones period. • Reading and writing large numbers should be related to numbers that have meanings (e.g., numbers found in the students' environment). Concrete materials, such as Base-10 blocks may be used to represent whole numbers through thousands. Larger numbers may be represented on place value charts. • Rounding is one of the estimation strategies that is often used to assess the reasonableness of a solution or to give an estimate of an amount. • Students should explore reasons for estimation, using practical experiences, and use rounding to solve practical situations. • The concept of rounding may be introduced through the use of a number line. When given a number to round, locate it on the number line. Next, determine the multiple of ten, hundred, or thousand it is between. Then identify to which it is closer. • A procedure for rounding numbers to the nearest ten, hundred, or thousand is as follows: – Look one place to the right of the digit to which you wish to round. – If the digit is less than 5, leave the digit in the rounding place as it is, and change the digits to the right of the rounding place to zero. – If the digit is 5 or greater, add 1 to the digit in the rounding place, and change the digits to the right of the rounding place to zero. • A procedure for comparing two numbers by examining may include the following: – Line up the numbers by place value by lining up the ones.

Number, Number Sense, Computation and Estimation		– Beginning at the left, find the first place value where the digits are different. – Compare the digits in this place value to determine which number is greater (or which is less). – Use the appropriate symbol > or < or the words *greater than* or *less than* to compare the numbers in the order in which they are presented. – If both numbers are the same, use the symbol = or the words *equal to*.
	3M-NSCE 2 (SOL 3.2)	• Addition and subtraction are inverse operations, as are multiplication and division. • In building thinking strategies for subtraction, an emphasis is placed on connecting the subtraction fact to the related addition fact. The same is true for division, where the division fact is tied to the related multiplication fact. Building fact sentences helps strengthen this relationship. • Addition and subtraction should be taught concurrently in order to develop understanding of the inverse relationship. • Multiplication and division should be taught concurrently in order to develop understanding of the inverse relationship.
	3M-NSCE 3 (SOL 3.3)	• A fraction is a way of representing part of a whole (as in a region/area model or a length/measurement model) or part of a group (as in a set model). Fractions are used to name a part of one thing or a part of a collection of things. Models can include pattern blocks, fraction bars, rulers, number line, etc. • In each area/region and length/measurement model, the parts must be equal-sized (congruent). Wholes are divided or partitioned into equal-sized parts. In the set model, each member of the set is an equal part of the set. The members of the set do not have to be equal in size. • The denominator tells how many equal parts are in the whole or set. The numerator tells how many of those parts are being considered. • Provide opportunities to make connections among fraction representations by connecting concrete or pictorial representations with oral language and symbolic representations. • Informal, integrated experiences with fractions at this level will help students develop a foundation for deeper learning at later grades. Understanding the language of fractions (e.g., *thirds* means "three equal parts of a whole," $\frac{1}{3}$ represents one of three equal-size parts when a pizza is shared among three students, or *three-fourths* means "three of four equal parts of a whole") furthers this development. • Comparing unit fractions (a fraction in which the numerator is one) builds a mental image of fractions and the understanding that as the number of pieces of a whole increases, the size of one single piece decreases (e.g., $\frac{1}{5}$ of a bar is smaller than $\frac{1}{4}$ of a bar). • Comparing fractions to a benchmark on a number line (e.g., close to 0, less than $\frac{1}{2}$, exactly $\frac{1}{2}$, greater than $\frac{1}{2}$, or close to 1) facilitates the comparison of fractions when using concrete materials or pictorial models.
	3M-NSCE 4 (SOL 3.4)	• Addition is the combining of quantities; it uses the following terms: *addend* → 423 *addend* → $\underline{+\ 246}$ *sum* → 669

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Subtraction is the inverse of addition; it yields the difference between two numbers and uses the following terms:
minuend → 7,698
subtrahend → <u>– 5,341</u>
difference → 2,357
An algorithm is a step-by-step method for computing. An example of an approach to solving problems is Polya’s four-step plan:

Understand: Retell the problem; read it twice; take notes; study the charts or diagrams; look up words and symbols that are new.
Plan: Decide what operation(s) and sequence of steps to use to solve the problem.
Solve: Follow the plan and work accurately. If the first attempt does not work, try another plan.
Look back: Does the answer make sense?

Knowing whether to find an exact answer or to make an estimate is learned through practical experiences in recognizing which is appropriate.
When an exact answer is required, opportunities to explore whether the answer can be determined mentally or must involve paper and pencil or calculators help students select the correct approach.
Determining whether an estimate is appropriate and using a variety of strategies to estimate requires experiences with problem situations involving estimation.
There are a variety of mental mathematics strategies for each basic operation, and opportunities to practice these strategies give students the tools to use them at appropriate times. For example, with addition, mental mathematics strategies include

Adding 9: add 10 and subtract 1; and
Making 10: for column addition, look for numbers that group together to make 10.

Using Base-10 materials to model and stimulate discussion about a variety of problem situations helps students understand regrouping and enables them to move from the concrete to the abstract. Regrouping is used in addition and subtraction algorithms.
Conceptual understanding begins with concrete experiences. Next, the children must make connections that serve as a bridge to the symbolic. One strategy used to make connections is representations, such as drawings, diagrams,tally marks, graphs, or written comments.

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<td>3M-NSCE 5
(SOL 3.5)</td>
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The development of computational fluency relies on quick access to number facts.
A certain amount of practice is necessary to develop fluency with computational strategies; however, the practice must be motivating and systematic if students are to develop fluency in computation, whether mental, with manipulative materials, or with paper and pencil.
Strategies to learn the multiplication facts through the twelves table include an understanding of multiples/skip counting, properties of zero and one as factors, pattern of nines, commutative property, and related facts.
In order to develop and use strategies to learn the multiplication facts through the twelves table, students should use concrete materials, hundred chart, and mental mathematics.
To extend the understanding of multiplication, three models may be used:

The equal-sets or equal-groups model lends itself to sorting a variety of concrete objects into equal groups and reinforces repeated addition or skip counting.

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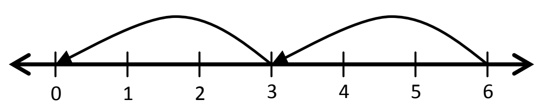
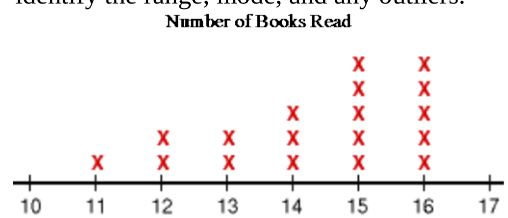
		– The array model, consisting of rows and columns (e.g., 3 rows of 4 columns for a 3-by-4 array) helps build the commutative property. – The length model (e.g., a number line) also reinforces repeated addition or skip counting.
	3M-NSCE 6 (SOL 3.6)	• The multiplication and division facts through the twelves tables should be modeled. • Multiplication is a shortcut for repeated addition. The terms associated with multiplication are listed below: *factor* → 54 *factor* → <u>× 3</u> *product* → 162 • Creating real-life problems and solving them facilitates the connection between mathematics and everyday experiences (e.g., area problems). • The use of Base-10 blocks and repeated addition can serve as a model. For example, 4 × 12 is read as four sets consisting of one rod and two units. The sum is renamed as four rods and eight units or 48. This can be thought of as 12 + 12 + 12 + 12 = (SET) • The use of Base-10 blocks and the array model can be used to solve the same problem. A rectangle array that is one rod and two units long by four units wide is formed. The area of this array is represented by 4 rods and 8 units. • The number line model can be used to solve a multiplication problem such as 3 × 4. This is represented on the number line by three jumps of four. • The number line model can be used to solve a division problem such as 6 ÷ 3 and is represented on the number line by noting how many jumps of three go from 6 to 0.  The number of jumps (two) of a given length (three) is the answer to the question. • An algorithm is a step-by-step method for computing.
	3M-NSCE 7 (SOL 3.7)	• A proper fraction is a fraction whose numerator is less than the denominator. A proper fraction is a fraction that is always less than one. • An improper fraction is a fraction whose numerator is greater than or equal to the denominator. An improper fraction is a fraction that is equal to or greater than one. • An improper fraction can be expressed as a mixed number. A mixed number is written as a whole number and a proper fraction. • The strategies of addition and subtraction applied to fractions are the same as the strategies applied to whole numbers. • Reasonable answers to problems involving addition and subtraction of fractions can be established by using benchmarks such as 0, $\frac{1}{2}$, and 1. For example, $\frac{3}{5}$ and $\frac{4}{5}$ are each greater than $\frac{1}{2}$, so their sum is greater than 1. • Concrete materials and pictorial models representing area/regions (circles, squares, and rectangles), length/measurements (fraction bars and strips), and sets (counters) can be used to add and subtract fractions having like denominators of 12 or less.

Measurement and Geometry	3M-MG 1 (SOL 3.8)	• The value of a collection of coins and bills can be determined by counting on, beginning with the highest value, and/or by grouping the coins and bills. • A variety of skills can be used to determine the change after a purchase, including – counting on, using coins and bills, i.e., starting with the amount to be paid (purchase price), counting forward to the next dollar, and then counting forward by dollar bills to reach the amount from which to make change; and – mentally calculating the difference.
	3M-MG 2 (SOL 3.9)	• Weight and mass are different. Mass is the amount of matter in an object. Weight is determined by the pull of gravity on the mass of an object. The mass of an object remains the same regardless of its location. The weight of an object changes dependent on the gravitational pull at its location. In everyday life, most people are actually interested in determining an object's mass, although they use the term *weight* (e.g., "How much does it weigh?" versus "What is its mass?"). • The concept of a standard measurement unit is one of the major ideas in understanding measurement. Familiarity with standard units is developed through hands-on experiences of comparing, estimating, measuring, and constructing. • Benchmarks of common objects need to be established for each of the specified units of measure (e.g., the mass of a mathematics book is about one kilogram). Practical experience measuring the mass of familiar objects helps to establish benchmarks and facilitates the student's ability to estimate measures. • One unit of measure may be more appropriate than another to measure an object, depending on the size of the object and the degree of accuracy desired. • Correct use of measurement tools is essential to understanding the concepts of measurement. • Perimeter is the distance around any two-dimensional figure and is found by adding the measures of the sides. Area is a two-dimensional measure and is therefore measured in square units. • Area is the number of square units needed to cover a figure, or more precisely, it is the measure in square units of the interior region of a two-dimensional figure.
	3M-MG 3 (SOL 3.11)	• While digital clocks make reading time easy, it is necessary to ensure that students understand that there are sixty minutes in an hour. • Use of a demonstration clock with gears ensures that the positions of the hour hand and the minute hand are precise when time is read**.** • Students need to understand that time has passed or will pass. • Elapsed time is the amount of time that has passed between two given times. • Elapsed time should be modeled and demonstrated using geared analog clocks and timelines**.** • It is necessary to ensure that students understand that there are sixty minutes in an hour when using analog and digital clocks. • Elapsed time can be found by counting on from the beginning time to the finishing time. – Count the number of whole hours between the beginning time and the finishing time. For example, to find the elapsed time between 7 a.m. and 10 a.m., students can count on to find the difference between the times (7 and 10), so the total elapsed time is 3 hours.

	3M-MG 4 (SOL 3.14)	• The van Hiele theory of geometric understanding describes how students learn geometry and provides a framework for structuring student experiences that should lead to conceptual growth and understanding. **Level 0: Pre-recognition.** Geometric figures are not recognized. For example, students cannot differentiate between three-sided and four-sided polygons. **Level 1: Visualization.** Geometric figures are recognized as entities, without any awareness of parts of figures or relationships between components of a figure. Students should recognize and name figures and distinguish a given figure from others that look somewhat the same (e.g., “I know it’s a rectangle because it looks like a door, and I know that the door is a rectangle.”). **Level 2: Analysis**. Properties are perceived, but are isolated and unrelated. Students should recognize and name properties of geometric figures (e.g., “I know it’s a rectangle because it’s closed, it has four sides and four right angles, and opposite sides are parallel.”). • A plane geometric figure is any two-dimensional closed figure. Circles and polygons are examples of plane geometric figures. • Three-dimensional figures are called solid figures or simply solids. Solids enclose a region of space. The interior of both plane and solid figures are not part of the figure. Solids are classified by the types of surfaces they have. These surfaces may be flat, curved, or both. • The study of geometric figures must be active, using visual images and concrete materials. • Access to a variety of concrete tools such as graph paper, pattern blocks, geoboards, and geometric solids is greatly enhanced by computer software tools that support exploration. • Opportunity must be provided for building and using geometric vocabulary to describe plane and solid figures. • A cube is a solid figure with six congruent square faces and with every edge the same length. A cube has 8 vertices and 12 edges. • A cylinder is a solid figure formed by two congruent parallel circles joined by a curved surface. • A cone is a solid, pointed figure that has a flat, round face (usually a circle) that is joined to a vertex by a curved surface. • A rectangular prism is a solid figure in which all six faces are rectangles with three pair of parallel congruent opposite faces. • A sphere is a solid figure with all of its points the same distance from its center. • A square pyramid is a solid figure with one square face and four triangular faces that share a common vertex. • A face is a polygon that serves as one side of a solid figure (e.g., a square is a face of a cube). • An angle is formed by two rays with a common endpoint. This endpoint is called the *vertex*. Angles are found wherever lines intersect. An angle can be named in three different ways by using – three letters to name, in this order, a point on one ray, the vertex, and a point on the other ray; – one letter at the vertex; or – a number written inside the rays of the angle. • An edge is the line segment where two faces of a solid figure intersect. • A vertex is the point at which two lines, line segments, or rays meet to form an angle. It is also the point on a three dimensional figure where three or more faces intersect. • Students should be reminded that a solid geometric object is hollow rather than solid. The “solid” indicates a three-dimensional figure.

Probability, Statistics, Patterns, Functions, and Algebra	3M-PSPFA 1 (SOL 3.17)	• Investigations involving data should occur frequently and relate to students' experiences, interests, and environment. • Formulating questions for investigations is student-generated at this level. For example: What is the cafeteria lunch preferred by students in the class when four lunch menus are offered? • The purpose of a graph is to represent data gathered to answer a question. • Bar graphs are used to compare counts of different categories (categorical data). Using grid paper ensures more accurate graphs. – A bar graph uses parallel, horizontal or vertical bars to represent counts for categories. One bar is used for each category, with the length of the bar representing the count for that category. – There is space before, between, and after the bars. – The axis displaying the scale representing the count for the categories should extend one increment above the greatest recorded piece of data. Third grade students should collect data that are recorded in increments of whole numbers, usually multiples of 1, 2, 5, or 10. – Each axis should be labeled, and the graph should be given a title. – Statements representing an analysis and interpretation of the characteristics of the data in the graph (e.g., similarities and differences, least and greatest, the categories, and total number of responses) should be written. • A line plot shows the frequency of data on a number line. Line plots are used to show the spread of the data and quickly identify the range, mode, and any outliers.  *Each x represents one student* • When data are displayed in an organized manner, the results of the investigations can be described and the posed question answered. • Recognition of appropriate and inappropriate statements begins at this level with graph interpretations.
	3M-PSPFA 2 (SOL 3.19)	• Exploring patterns requires active physical and mental involvement. • The use of materials to extend patterns permits experimentation or trial-and-error approaches that are almost impossible without them. • Reproduction of a given pattern in a different representation, using symbols and objects, lays the foundation for writing numbers symbolically or algebraically. • The simplest types of patterns are repeating patterns. In each case, students need to identify the basic unit of the pattern and repeat it. Opportunities to create, recognize, describe, and extend repeating patterns are essential to the primary school experience. • Growing patterns are more difficult for students to understand than repeating patterns because not only must they determine what comes next, they must also begin the process of generalization. Students need experiences with growing patterns in both arithmetic and geometric formats.

		• Create an arithmetic number pattern. Sample numeric patterns include – 6, 9, 12, 15, 18, …(growing pattern); – 1, 2, 4, 7, 11, 16, …(growing pattern); – 20, 18, 16, 14,…(growing pattern); and – 1, 3, 5, 1, 3, 5, 1, 3, 5 …(repeating pattern). • In geometric patterns, students must often recognize transformations of a figure, particularly rotation or reflection. Rotation is the result of turning a figure around a point or a vertex, and reflection is the result of flipping a figure over a line. • Sample geometric patterns include – ∆ O O ∆ ∆ O O O ∆ ∆ ∆ …; and – □□★★□★□□★★□★…. • A table of values can be analyzed to determine the pattern that has been used, and that pattern can then be used to find the next value.
	3M-PSPFA 3 (SOL 3.20)	• Investigating arithmetic operations with whole numbers helps students learn about several different properties of arithmetic relationships. These relationships remain true regardless of the numbers. • The commutative property for addition states that changing the order of the addends does not affect the sum (e.g., $4 + 3 = 3 + 4$). Similarly, the commutative property for multiplication states that changing the order of the factors does not affect the product (e.g., $2 \times 3 = 3 \times 2$). • The identity property for addition states that if zero is added to a given number, the sum is the same as the given number. The identity property of multiplication states that if a given number is multiplied by one, the product is the same as the given number. • A number sentence is an equation with numbers (e.g., $6 + 3 = 9$; or $6 + 3 = 4 + 5$).