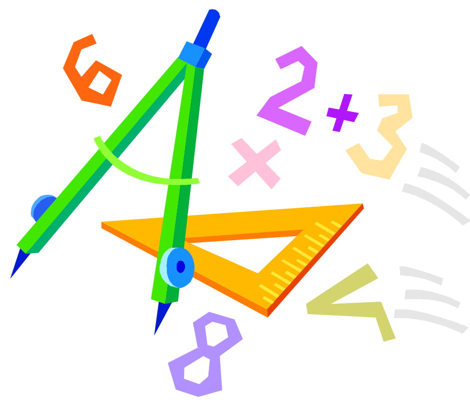
MATHEMATICS
ALIGNED STANDARDS OF LEARNING
CURRICULUM FRAMEWORK
GRADE 6



6M-NSCE1 The student will
a) demonstrate a simple ratio relationship.

UNDERSTANDING THE STANDARD (Background Information for Instructor Use Only)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• A ratio is a comparison of any two quantities. A ratio is used to represent relationships within and between sets. • A ratio can compare part of a set to the entire set (part-whole comparison). • A ratio can compare part of a set to another part of the same set (part-part comparison). • A ratio can compare part of a set to a corresponding part of another set (part-part comparison). • A ratio can compare all of a set to all of another set (whole-whole comparison). • The order of the quantities in a ratio is directly related to the order of the quantities expressed in the relationship. For example, if asked for the ratio of the number of cats to dogs in a park, the ratio must be expressed as the number of cats to the number of dogs, in that order. • A ratio is a multiplicative comparison of two numbers, measures, or quantities. • All fractions are ratios and vice versa. • Ratios may or may not be written in simplest form. • Ratios can compare two parts of a whole. • Rates can be expressed as ratios.	• What is a ratio? A ratio is a comparison of any two quantities. A ratio is used to represent relationships within a set and between two sets. A ratio can be written using fraction form ($\frac{2}{3}$), a colon (2:3), or the word *to* (2 to 3).	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Describe a relationship within a set by comparing part of the set to the entire set. • Describe a relationship between two sets by comparing part of one set to a corresponding part of the other set. • Describe a relationship between two sets by comparing all of one set to all of the other set. • Describe a relationship within a set by comparing one part of the set to another part of the same set. • Represent a relationship in words that makes a comparison by using the notations $\frac{a}{b}$, a:b, and *a to b*. • Create a relationship in words for a given ratio expressed symbolically.

6M-NSCE2 The student will

a) understand that positive and negative numbers are used together to describe quantities having opposite directions or values (e.g., temperature above/below zero).

UNDERSTANDING THE STANDARD (Background Information for Instructor Use Only)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• Integers are the set of whole numbers, their opposites, and zero. • Positive integers are greater than zero. • Negative integers are less than zero. • Zero is an integer that is neither positive nor negative. • A negative integer is always less than a positive integer. • When comparing two negative integers, the negative integer that is closer to zero is greater. • An integer and its opposite are the same distance from zero on a number line. For example, the opposite of 3 is -3. • The absolute value of a number is the distance of a number from zero on the number line regardless of direction. Absolute value is represented as $\|-6\| = 6$. • On a conventional number line, a smaller number is always located to the left of a larger number (e.g., –7 lies to the left of –3, thus –7 < –3; 5 lies to the left of 8 thus 5 is less than 8).	• What role do negative integers play in practical situations? Some examples of the use of negative integers are found in temperature (below 0), finance (owing money), below sea level. There are many other examples. • How does the absolute value of an integer compare to the absolute value of its opposite? They are the same because an integer and its opposite are the same distance from zero on a number line.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Identify an integer represented by a point on a number line. • Represent integers on a number line. • Order and compare integers using a number line. • Compare integers, using mathematical symbols (<, >, =). • Identify and describe the absolute value of an integer.

6M-NSCE3 The student will

a) compare the relationships between two unit fractions.

UNDERSTANDING THE STANDARD (Background Information for Instructor Use Only)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• Using manipulatives to build conceptual understanding and using pictures and sketches to link concrete examples to the symbolic enhance students' understanding of operations with fractions and help students connect the meaning of whole number computation to fraction computation. • Multiplication and division of fractions can be represented with arrays, paper folding, repeated addition, repeated subtraction, fraction strips, pattern blocks and area models. • When multiplying a whole by a fraction such as 3 x $\frac{1}{2}$, the meaning is the same as with multiplication of whole numbers: 3 groups the size of $\frac{1}{2}$ of the whole. • When multiplying a fraction by a fraction such as $\frac{2}{3}\cdot\frac{3}{4}$, we are asking for part of a part. • When multiplying a fraction by a whole number such as $\frac{1}{2}$ x 6, we are trying to find a part of the whole. • For measurement division, the divisor is the number of groups. You want to know how many are in each of those groups. Division of fractions can be explained as how many of a given divisor are needed to equal the given	• When multiplying fractions, what is the meaning of the operation? When multiplying a whole by a fraction such as 3 x $\frac{1}{2}$, the meaning is the same as with multiplication of whole numbers: 3 groups the size of $\frac{1}{2}$ of the whole. When multiplying a fraction by a fraction such as $\frac{2}{3}\cdot\frac{3}{4}$, we are asking for part of a part. When multiplying a fraction by a whole number such as $\frac{1}{2}$ x 6, we are trying to find a part of the whole. • What does it mean to divide with fractions? For measurement division, the divisor is the number of groups and the quotient will be the number of groups in the dividend. Division of fractions can be explained as how many of a given divisor are needed to equal the given dividend. In other words, for $\frac{1}{4}\div\frac{2}{3}$ the question is, "How many	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Select appropriate methods and tools from among paper and pencil, estimation, mental computation, and calculators according to the context and nature of the computation in order to compute with whole numbers. • Create single-step and multistep problems involving the operations of addition, subtraction, multiplication, and division with and without remainders of whole numbers, using practical situations. • Estimate the sum, difference, product, and quotient of whole number computations. • Solve single-step and multistep problems involving addition, subtraction, multiplication, and division with and without remainders of whole numbers, using paper and pencil, mental computation, and calculators in which – sums, differences, and products will not exceed five digits; – multipliers will not exceed two digits; – divisors will not exceed two digits; or – dividends will not exceed four digits. • Use two or more operational steps to solve a multistep problem. Operations can be the same or different.

6M-NSCE3 The student will

a) compare the relationships between two unit fractions.

UNDERSTANDING THE STANDARD (Background Information for Instructor Use Only)	**ESSENTIAL UNDERSTANDINGS**	**ESSENTIAL KNOWLEDGE AND SKILLS**
dividend. In other words, for $\frac{1}{4} \div \frac{2}{3}$, the question is, "How many $\frac{2}{3}$ make $\frac{1}{4}$?" • For partition division the divisor is the size of the group, so the quotient answers the question, "How much is the whole?" or "How much for one?"	$\frac{2}{3}$ make $\frac{1}{4}$?" For partition division the divisor is the size of the group, so the quotient answers the question, "How much is the whole?" or "How much for one?"	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Demonstrate multiplication and division of fractions using multiple representations. • Model algorithms for multiplying and dividing with fractions using appropriate representations.

6M-NSCE4 The student will

a) solve two factor multiplication problems with products up to 50 using concrete objects and/or calculators.

UNDERSTANDING THE STANDARD (Background Information for Instructor Use Only)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• Different strategies can be used to estimate the result of computations and judge the reasonableness of the result. For example: What is an approximate answer for 2.19 ÷ 0.8? The answer is around 2 because 2 ÷ 1 = 2. • Understanding the placement of the decimal point is very important when finding quotients of decimals. Examining patterns with successive decimals provides meaning, such as dividing the dividend by 6, by 0.6, by 0.06, and by 0.006. • Solving multistep problems in the context of real-life situations enhances interconnectedness and proficiency with estimation strategies. • Examples of practical situations solved by using estimation strategies include shopping for groceries, buying school supplies, budgeting an allowance, deciding what time to leave for school or the movies, and sharing a pizza or the prize money from a contest.	• What is the role of estimation in solving problems? Estimation gives a reasonable solution to a problem when an exact answer is not required. If an exact answer is required, estimation allows you to know if the calculated answer is reasonable.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Solve single-step and multistep practical problems involving addition, subtraction, multiplication and division with decimals expressed to thousandths with no more than two operations.

6M-NSCE5 The student will
a) identify equivalent number sentences.

UNDERSTANDING THE STANDARD (Background Information for Instructor Use Only)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• The order of operations is a convention that defines the computation order to follow in simplifying an expression. • The order of operations is as follows: – First, complete all operations within grouping symbols*. If there are grouping symbols within other grouping symbols, do the innermost operation first. – Second, evaluate all exponential expressions. – Third, multiply and/or divide in order from left to right. – Fourth, add and/or subtract in order from left to right. * Parentheses ( ), brackets [ ], braces {}, and the division bar – as in $\frac{3+4}{5+6}$ should be treated as grouping symbols. • The power of a number represents repeated multiplication of the number (e.g., $8^3 = 8 \cdot 8 \cdot 8$). The base is the number that is multiplied, and the exponent represents the number of times the base is used as a factor. In the example, 8 is the base, and 3 is the exponent. • Any number, except 0, raised to the zero power is 1. Zero to the zero power is undefined.	• What is the significance of the order of operations? The order of operations prescribes the order to use to simplify expressions containing more than one operation. It ensures that there is only one correct answer.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Simplify expressions by using the order of operations in a demonstrated step-by-step approach. The expressions should be limited to positive values and not include braces { } or absolute value | |. • Find the value of numerical expressions, using order of operations, mental mathematics, and appropriate tools. Exponents are limited to positive values.

6M-MG1 The student will
- **a) demonstrate area;**
- **b) identify common three-dimensional shapes.**

UNDERSTANDING THE STANDARD (Background Information for Instructor Use Only)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• Experiences in deriving the formulas for area, perimeter, and volume using manipulatives such as tiles, one-inch cubes, adding machine tape, graph paper, geoboards, or tracing paper, promote an understanding of the formulas and facility in their use.† • The perimeter of a polygon is the measure of the distance around the polygon. • Circumference is the distance around or perimeter of a circle. • The area of a closed curve is the number of nonoverlapping square units required to fill the region enclosed by the curve. • The perimeter of a square whose side measures s is 4 times s ($P = 4s$), and its area is side times side ($A = s^2$). • The perimeter of a rectangle is the sum of twice the length and twice the width [$P = 2l + 2w$, or $P = 2(l + w)$], and its area is the product of the length and the width ($A = lw$). • The value of pi (π) is the ratio of the circumference of a circle to its diameter. • The ratio of the circumference to the diameter of a circle is a constant value, pi (π), which can be approximated by measuring various sizes of circles.	• What is the relationship between the circumference and diameter of a circle? The circumference of a circle is about 3 times the measure of the diameter. • What is the difference between area and perimeter? Perimeter is the distance around the outside of a figure while area is the measure of the amount of space enclosed by the perimeter. • What is the relationship between area and surface area? Surface area is calculated for a three-dimensional figure. It is the sum of the areas of the two-dimensional surfaces that make up the three-dimensional figure.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Derive an approximation for pi (3.14 or $\frac{22}{7}$) by gathering data and comparing the circumference to the diameter of various circles, using concrete materials or computer models. • Find the circumference of a circle by substituting a value for the diameter or the radius into the formula $C = \pi d$ or $C = 2\pi r$. • Find the area of a circle by using the formula $A = \pi r^2$. • Apply formulas to solve practical problems involving area and perimeter of triangles and rectangles. • Create and solve problems that involve finding the circumference and area of a circle when given the diameter or radius. • Solve problems that require finding the surface area of a rectangular prism, given a diagram of the prism with the necessary dimensions labeled. • Solve problems that require finding the volume of a rectangular prism given a diagram of the prism with the necessary dimensions labeled.

6M-MG1 The student will

a) demonstrate area;

b) identify common three-dimensional shapes.

UNDERSTANDING THE STANDARD (Background Information for Instructor Use Only)	**ESSENTIAL UNDERSTANDINGS**	**ESSENTIAL KNOWLEDGE AND SKILLS**
• The fractional approximation of pi generally used is $\frac{22}{7}$. • The decimal approximation of pi generally used is 3.14. • The circumference of a circle is computed using $C = \pi d$ or $C = 2\pi r$, where *d* is the diameter and *r* is the radius of the circle. • The area of a circle is computed using the formula $A = \pi r^2$, where *r* is the radius of the circle. • The surface area of a rectangular prism is the sum of the areas of all six faces ($SA = 2lw + 2lh + 2wh$). • The volume of a rectangular prism is computed by multiplying the area of the base, B, (length x width) by the height of the prism ($V = lwh = Bh$).		

6M-PSPFA1 The student will

a) display data on a graph or table that shows variability in the data;
b) summarize data distributions on a graph or table;
c) answer a question related to the collected data from an experiment, given a model of data, or from data collected by the student.

UNDERSTANDING THE STANDARD (Background Information for Instructor Use Only)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• To collect data for any problem situation, an experiment can be designed, a survey can be conducted, or other data-gathering strategies can be used. The data can be organized, displayed, analyzed, and interpreted to answer the problem. • Different types of graphs are used to display different types of data. – Bar graphs use categorical (discrete) data (e.g., months or eye color). – Line graphs use continuous data (e.g., temperature and time). – Circle graphs show a relationship of the parts to a whole. • All graphs include a title, and data categories should have labels. • A scale should be chosen that is appropriate for the data. • A key is essential to explain how to read the graph. • A title is essential to explain what the graph represents. • Data are analyzed by describing the various features and elements of a graph.	• What types of data are best presented in a circle graph? Circle graphs are best used for data showing a relationship of the parts to the whole.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Collect, organize and display data in circle graphs by depicting information as fractional. • Draw conclusions and make predictions about data presented in a circle graph. • Compare and contrast data presented in a circle graph with the same data represented in other graphical forms.

6M-PSPFA2 The student will

a) match an equation to a real-world problem in which variables are used to represent numbers.

UNDERSTANDING THE STANDARD (Background Information for Instructor Use Only)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• A one-step linear equation is an equation that requires one operation to solve. • A mathematical expression contains a variable or a combination of variables, numbers, and/or operation symbols and represents a mathematical relationship. An expression cannot be solved. • A term is a number, variable, product, or quotient in an expression of sums and/or differences. In $7x^2 + 5x – 3$, there are three terms, $7x^2$, $5x$, and 3. • A coefficient is the numerical factor in a term. For example, in the term $3xy^2$, 3 is the coefficient; in the term z, 1 is the coefficient. • Positive rational solutions are limited to whole numbers and positive fractions and decimals. • An equation is a mathematical sentence stating that two expressions are equal. • A variable is a symbol (placeholder) used to represent an unspecified member of a set.	• When solving an equation, why is it necessary to perform the same operation on both sides of an equal sign? To maintain equality, an operation performed on one side of an equation must be performed on the other side.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections and representation to** • Represent and solve a one-step equation, using a variety of concrete materials such as colored chips, algebra tiles, or weights on a balance scale. • Solve a one-step equation by demonstrating the steps algebraically. • Identify and use the following algebraic terms appropriately: *equation*, *variable, expression*, *term*, and *coefficient*.

6M-PSPFA3 The student will
a) demonstrate understanding of equivalent expressions.

UNDERSTANDING THE STANDARD (Background Information for Instructor Use Only)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• Identity elements are numbers that combine with other numbers without changing the other numbers. The additive identity is zero (0). The multiplicative identity is one (1). There are no identity elements for subtraction and division. • The additive identity property states that the sum of any real number and zero is equal to the given real number (e.g., 5 + 0 = 5). • The multiplicative identity property states that the product of any real number and one is equal to the given real number (e.g., $8 \cdot 1 = 8$). • Inverses are numbers that combine with other numbers and result in identity elements. • The multiplicative inverse property states that the product of a number and its multiplicative inverse (or reciprocal) always equals one (e.g., $4 \cdot \frac{1}{4} = 1$). • Zero has no multiplicative inverse. • The multiplicative property of zero states that the product of any real number and zero is zero. • Division by zero is not a possible arithmetic operation. Division by zero is undefined.	• How are the identity properties for multiplication and addition the same? Different? For each operation the identity elements are numbers that combine with other numbers without changing the value of the other numbers. The additive identity is zero (0). The multiplicative identity is one (1). • What is the result of multiplying any real number by zero? The product is always zero. • Do all real numbers have a multiplicative inverse? No. Zero has no multiplicative inverse because there is no real number that can be multiplied by zero resulting in a product of one.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Identify a real number equation that represents each property of operations with real numbers, when given several real number equations. • Test the validity of properties by using examples of the properties of operations on real numbers. • Identify the property of operations with real numbers that is illustrated by a real number equation. NOTE: The commutative, associative and distributive properties are taught in previous grades.