College and Career Readiness Standards for Mathematics

Orchestrating Mathematical Discourse to Enhance Student Learning

Creating successful classroom environments where every student participates in rigorous discussions

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Gladis is a respected scholar of mathematics education. She is known for her research on mathematics education, middle-grades mathematics teaching and learning, and the preparation of teachers of mathematics. She has authored numerous publications on topics such as teacher education, the effective teaching of at-risk students, and the use of technology in teaching and learning.

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Her work is driven by the belief that all students deserve equitable access to mathematics so that they can fully benefit from the options offered by education.

Ready: Raising the bar and making it reachable

With the *Ready* programs for Reading, Mathematics, and now Writing—all built from scratch to meet the increased expectations of the Common Core State Standards—Curriculum Associates is providing resources that fully prepare students to be successful with the new, more rigorous standards while providing teachers the point-of-use instructional support to teach them most effectively.

Ready Mathematics is a rigorous instruction and practice program that supports a rich classroom environment in which mathematical reasoning, mathematical discourse, and mathematical practices all thrive. Its comprehensive teacher support makes it powerfully simple for teachers to implement.

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Introduction: Understanding the Importance of Mathematical Discourse

The importance of engaging students in meaningful mathematical discussion has long been identified as an **essential component of students' mathematics learning** *(National Council of Teachers of Mathematics, 1991, 2000, 2007, 2014)*. When students share and exchange their ideas, both they and their teachers benefit.

Students can reflect on their own understanding while making sense of and critiquing the ideas of others in a **collaborative and supportive learning environment,** as required by the Common Core Standards for Mathematical Practice. They can make conjectures, link prior knowledge to current understanding, reason about mathematics, refine and amend their approaches, and **take ownership** of their mathematical knowledge. Students can learn to use the **tools of mathematical discourse**—including words, symbols, diagrams, physical models, and technology—to present and defend their ideas. Teachers benefit too, because they have the opportunity to access, monitor, and evaluate students' mathematical understanding and development.

"The discourse of a classroom—the ways of representing, thinking, talking, agreeing, and disagreeing—is central to what and how students learn mathematics" (NCTM, 2007, p. 46).

Mathematical discussion is viewed as "a primary mechanism for developing conceptual understanding and meaningful learning of mathematics" (NCTM, 2014, p. 30).

This vignette illustrates some of the classroom techniques used to facilitate students' engagement in classroom discourse.

A Classroom Example of Mathematical Discourse

Teacher: In a fraction, what does the denominator—the bottom number—tell us?

Julie: The total number of parts or sections.

Teacher: *[to the class]* Using thumbs up or thumbs down, show us if you agree with Julie.

[after observing the responses] Some of you disagree. Would someone explain why? Okay, James, please explain.

James: She said the denominator tells the total number of parts, but all the parts have to be the same size.

Teacher: Thank you, James. Can someone else say that differently?

Nicole: They have to be the same.

Teacher: Can you say that again using some of the math words we've been learning?

Nicole: The bottom number, I mean, the denominator tells us the total number of equal-sized parts.

Teacher: Thank you, Nicole.

In order to review the information needed for the lesson, the teacher could have simply told the students what she wanted them to know about the denominator. For example, she could have said, "Given a whole, the denominator tells you the number of equal-sized parts." But she structured the lesson to encourage student interaction, address gaps in student understanding, and help students express mathematical concepts more precisely.

Reflecting on the importance of orchestrating mathematical discourse

- How did these students benefit by sharing what they knew as part of the mathematics discussion?
- Why did the teacher allow students to evaluate the correctness of Julie's answer?
- Why did the teacher ask another student to restate what James said?

Establishing a Discourse-Rich Mathematics Learning Community

To engage students in productive mathematics discussions, it is important to establish a learning environment that **welcomes student involvement**. The first step is establishing the expectation that every student will contribute to the discourse community.

Students must be encouraged to use their problem-solving, reasoning, and communication skills to make conjectures, explore their own ideas and approaches, and find solutions to routine and non-routine mathematics problems.

In particular, the five process standards—problem solving, reasoning and proof, communication, connections, and representation (*NCTM, 2000*)—can be seen in action in a discourse-rich mathematics community as students interact, question one another, and convey their understanding. As a result, teachers can recognize and reinforce errors as natural occurrences that enhance learning.

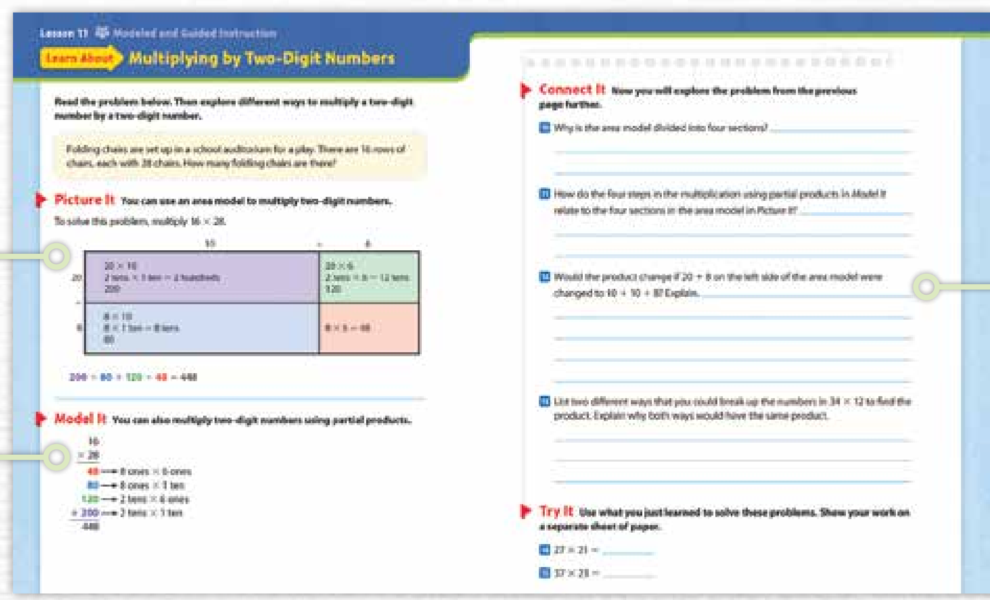
A visitor to a classroom rich with mathematical sense making should be able to identify the following traits:

- Students and teachers **acknowledge and discuss errors and the reasons behind them** so that students build greater understanding.
- Students **question each other using mathematics arguments** to establish the correctness of solutions.
- Students **reach and justify conclusions based on their own mathematics knowledge** without relying on the authority of teachers.
- Students **engage in "productive struggle"** with appropriate scaffolds for support.

Students must be encouraged to use a variety of approaches to convey their knowledge and solution strategies, including oral presentations; written explanations; and physical, graphical, pictorial, or symbolic representations.

Carefully crafted problems and questions, like these from Ready Mathematics *Instruction, help students understand problems more deeply, make connections between representations, and communicate problem-solving strategies.*

Students need to see a variety of approaches and solution strategies.



Questions engage students in discourse and require them to explain concepts and critique the reasoning of others.

Engaging Every Student in Mathematical Discourse

All students are mathematics language learners, regardless of their level of English language proficiency, and whether they are English Language Learners (ELLs) or proficient speakers of English (*Thompson et al., 2008*).

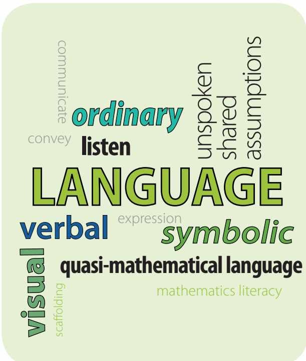
Although it is important to build scaffolds to support students who are learning English and mathematics at the same time, it is essential to recognize that mathematics is a technical language that all students must learn. Students who are English speakers also require support as they learn the language of mathematics. Mathematical discourse allows all students to develop mathematical language.

It is essential to recognize that ***mathematics is a technical language*** *that all students must learn.*

As indicated in the Standards for Mathematical Practice, students should attend to precision by using correct mathematical language, including vocabulary, symbolic representations, syntax, semantics, and linguistic features (*Kersaint, Thompson, and Petkova, 2013*). In addition, they must have **ample opportunities to use the language of mathematics** as they engage in various forms of communication. Discourse allows students to practice precision in multiple areas, including:

1. **Vocabulary**
   - English words with a different meaning in mathematics, such as "negative," "table," or "rational."
   - Specialized terms, such as "hypotenuse" or "trapezoid."
   - Terms with multiple meanings in mathematics, such as "median" or "base."
2. **Symbols**
   - Ways to read and interpret symbolic representations. *For example,* "a × b" *can be expressed as* "a *times* b," "*the product of* a *and* b," *or* "*multiply* a *and* b."
3. **Syntax**
   - Understanding the rules that govern the structure of sentences. *For example, "A number* y *is 4 more than a number* x*" is translated symbolically to* "y = x + 4."
4. **Semantics**
   - The process of making meaning from language.
   - The same mathematics word may be interpreted differently depending on the context. *For example, the median of a set of numbers versus the median of a triangle.*

Recognizing Different Approaches for Communicating Mathematical Knowledge



Teachers can use numerous techniques to encourage students to learn the language of mathematics and engage in mathematical discourse.

There are various means for communicating mathematics knowledge, which collectively represent an indication of mathematics literacy. They include listening, speaking, reading, writing, and interpreting information. It is through these various means of communicating that teachers uncover what a student knows or understands. Students may express their mathematics knowledge with different levels of sophistication. It is important for teachers to recognize the various ways in which students may attempt to convey what they know, honor what they know and how it is represented, and provide scaffolding support to move students from less- to more-sophisticated and -precise uses of mathematics language.

Pirie *(1998)* identifies six means of communicating verbally or in writing in the mathematics classroom. Mathematical discourse may involve one or more of these means of communication:

1. **Ordinary language**
   Students use their informal, non-academic vocabulary and language skills to convey ideas.

2. **Mathematics verbal language**
   Students use mathematics vocabulary as they speak or write. For example, "*The square root of 16 is 4.*"

3. **Symbolic language**
   Students express ideas using mathematical symbols. For example, they write "$x < 3$" to mean that the value of x is less than 3.

4. **Visual representations**
   Students use concrete models or images such as pictures, tables, or graphs to convey mathematical ideas.

5. **Unspoken shared assumptions**
   For example, in the problem "How much sod is needed for a plot of land that is 8 feet long and 6 feet wide?", the listener or reader recognizes that "sod" is dirt with grass growing in it that is used to cover land.

6. **Quasi-mathematical language**
   Students' communication is incoherent because they lack the necessary vocabulary, understanding, or language skills, although they may demonstrate nuggets of understanding.

   Students who use quasi-mathematical language can be helped by hearing others express themselves with precision and by receiving guidance in developing their mathematical-language skills.

Understanding Student and Teacher Roles in Mathematical Discourse

Teachers must prepare students to engage in mathematics discussions. Depending on prior experiences, students might find the expectation to engage in discussions in a mathematics class foreign.

To support students, teachers must help students create a vision for expected behaviors and actions, prepare them for their roles by modeling or role-playing, and reinforce these behaviors consistently. It is only through these actions that students learn how to function as members of a discourse-rich mathematics community.

Table 1.

Facilitate meaningful mathematical discourse

Teacher and student actions

What are teachers doing?	What are students doing?
Engaging students in purposeful sharing *of mathematical ideas, reasoning, and approaches, using varied representations.*	***Presenting and explaining*** *ideas, reasoning, and representations to one another in pairs, small groups, or whole-class discourse.*
Selecting and sequencing student approaches and solution strategies *for whole-class analysis and discussion.*	***Listening carefully to and critiquing the reasoning of peers****, using examples to support or counterexamples to refute arguments.*
Facilitating discourse among students *by positioning them as authors of ideas, who explain and defend their approaches.*	***Seeking to understand the approaches used by peers*** *by asking clarifying questions, trying out others' strategies, and describing the approaches used by others.*
Ensuring progress toward mathematical goals *by making explicit connections to student approaches and reasoning.*	***Identifying how different approaches to solving a task are the same and how they are different.***

Source: National Council of Teachers of Mathematics (2014). Principles to Actions: Ensuring Mathematical Success for All. Reston, VA: The author (p. 355).

Teachers **monitor progress** as students engage in mathematics discussions. They support students as their mathematical knowledge grows and they become more skilled at expressing their ideas coherently and using vocabulary, syntax, and semantics precisely.

Teachers observe, listen to, and monitor students to **support instructional decision making (Figure 1)**. They determine what questions to ask, which students to call on, when to intervene, and when to extend student thinking.

For teachers, these means of communication provide opportunities to **understand student thinking and assess student knowledge.**

Mathematical discourse serves as a forum in which to exchange and record mathematics ideas, document evidence of learning, and monitor growth in knowledge development. Teachers gain insights not only about what students know but also about the approaches they use, how—and how well—they understand the ideas, and the ways they present their knowledge.

In addition to content knowledge, mathematical discourse allows teachers to monitor students' dispositions to gauge their developing confidence, interest, and perseverance.

Teachers must acknowledge students' communication efforts while also taking advantage of the opportunity to deepen their mathematical understanding and build their mathematics language skills.

In addition to content knowledge, mathematical discourse allows teachers to monitor students' dispositions to gauge their developing confidence, interest, and perseverance. Teachers can use this information to determine areas of confusion or frustration in order to decide when an intervention might be needed. They also examine understandings and misconceptions revealed during classroom discussions and adjust their lesson plans accordingly.

Teachers carefully consider the best ways to organize and coordinate student interaction in pairs, small groups, or whole-class interactions to maximize learning.

Teachers can ask questions from Figure 2 to direct and redirect student focus.

Figure 1.

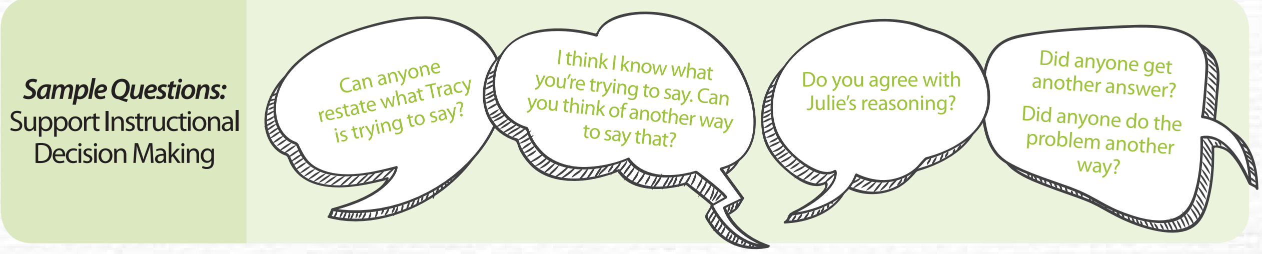
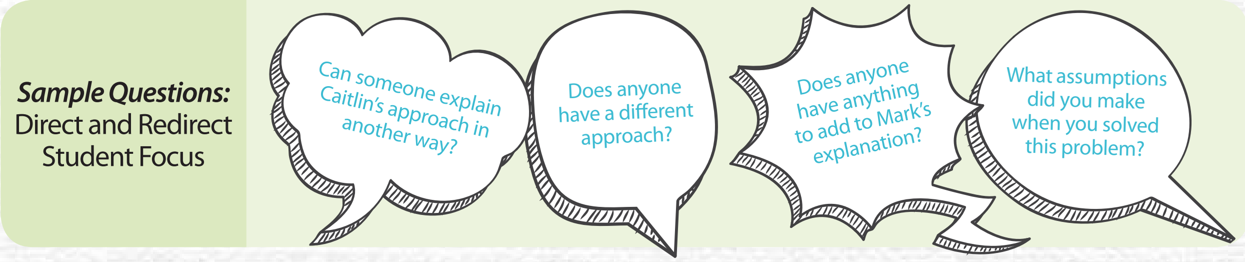
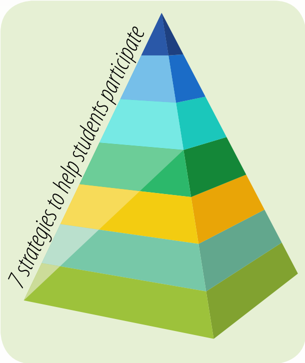


Figure 2.



Establishing Classroom Environments that Support Mathematical Discourse



It is important to establish learning environments and strategies that encourage student engagement in productive mathematics conversations. Teachers must **model the behavior** they want students to use when they work with peers, demonstrate ways to share solutions and strategies, establish expectations for mathematical discourse by clearly outlining the roles and expectations for "**productive talk**" in the mathematics classroom, and ensure that all students have **equal access** and opportunities to contribute to the mathematical discourse learning community. Using **effective strategies** like those listed below will support students as they learn to become participants in mathematical discourse.

Strategy 1: Help students work with and rely on one another

Rules such as "Ask three before you ask me" can help establish classroom expectations by encouraging students to seek assistance from peers before defaulting to the teacher. The teacher can also designate **student experts** (students who have demonstrated depth of understanding about a particular problem, concept, or procedure) whom other students can consult before approaching the teacher.

Strategy 2: Allow students to work independently before sharing in small or large groups

Students need time to gather their thoughts and identify what they know or do not know before they are exposed to the influence of other students. Then they can compare and contrast their approaches and solutions with those shared by others during the mathematics discussion. If they are new to the learning discourse community, it is important to coach them through the process so they know what is expected and feel secure in their attempts to contribute, even if their solutions are incorrect.

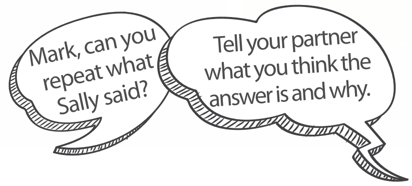
What is a student expert?

The definition of a student expert is not the same as the traditional definition of a good mathematics student. Any student who understands a particular topic, operation, concept, definition, or representation well can be a student expert. His or her expertise is tied to specific knowledge or ability, so all students have the opportunity to be experts if the teacher intentionally identifies their expertise to their peers.

Teachers can prepare students who are typically identified as low achievers, such as ELLs or members of underrepresented groups, to become student experts. They can do so by giving these students additional support and individualized practice time, ensuring they will have sufficient knowledge and understanding to be considered an expert on some aspect of mathematics. This is important because such students are seldom given opportunities to experience success or contribute to the learning of others in an active discourse-rich mathematics learning community.

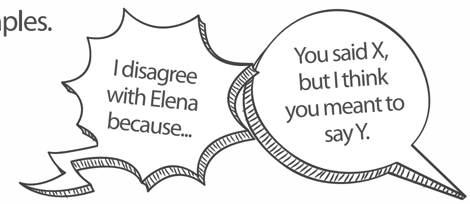
Strategy 3: Teachers can use questions and prompts to:

Encourage students to listen to each other.

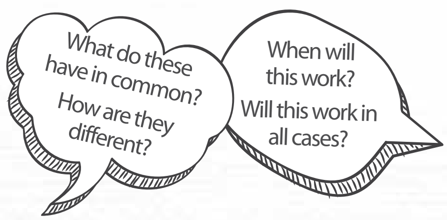


Ask students to clarify or restate peers' comments.

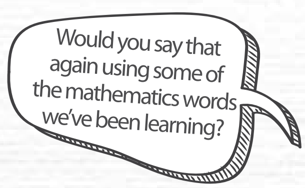
Teach students to respectfully critique the reasoning of their peers and disagree amicably. Give them some examples.



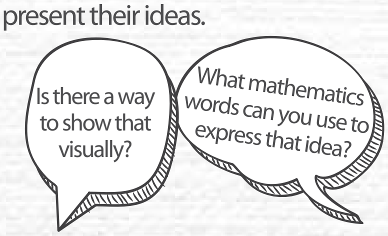
Compare and contrast different solution strategies.



Use the language of mathematics.



Use the tools of mathematical discourse to help students present their ideas.



Strategy 4: Use questions strategically to engage students in mathematical discourse

Teachers can engage students in mathematical discourse by posing questions that encourage discussion and debate and that require students to attend to particular aspects of the learning process and explain and justify their thinking.

Table 2, on pages 18–19, lists some questions teachers can use as they orchestrate whole-class discussions or monitor progress when students are working independently or in small groups. The questions are organized according to the intended student behaviors and outcomes.

Below are a few examples of questions to use in the classroom that promote mathematical thinking and discourse. For the entire list, please turn to page 18.

Questions that promote mathematical thinking and discourse help students...

...work together to make sense of mathematics:
"What strategy did you use?"

...rely more on themselves to determine whether something is mathematically correct:
"Is this a reasonable answer?"

...learn to reason mathematically:
"How did you begin to think about this problem?"

...learn to conjecture, invent, and solve problems:
"Do you see a pattern?"

...learn to connect mathematics, its ideas, and its applications:
"What is the relationship of this to that?"

...with problem comprehension:
"Would you please explain that in your own words?"

...focus on the mathematics from activities:
"What was one thing you learned (or two, or more)?"

...persevere:
"Have you tried making a guess?"

...evaluate their own processes, activities, and program:
"What have you accomplished?"

(A complete list of questions can be found on pp. 18–19.)

What is equal access?
Teachers must use a variety of classroom organizational structures to ensure that all students participate equally in the discourse community. Inevitably, some students will be more willing or more reluctant to participate than others. To make discourse equitable, teachers must engineer opportunities for all students to engage in equitable ways by monitoring and organizing their participation.

Strategy 5: Acknowledge the importance of mistakes in learning and understanding

- Recognize that students will make errors because they are exploring and making conjectures.
- Remind students constantly that errors are expected and natural and that they can be a good thing because they lead to enhanced learning.
- Help students recognize what they have learned by analyzing their mistakes and identifying misunderstandings.
- Allow students to ask questions to clarify and critique the reasoning of others.
- Encourage students to question the thinking of peers using mathematics dialogue to establish the correctness of solutions.
- Empower students to reach and justify conclusions based on their own mathematics knowledge without relying on the authority of the teacher.

Strategy 6: Use collaborative learning strategies to support students in preparation for whole-class discussions

When students work with peers or in small groups, they are able to take risks and build confidence on a small scale before they present solutions to the whole class. Teachers typically bring students with similar or diverse abilities together in small groups so that they can learn from each other. Small-group strategies that can help students become more comfortable with mathematical discourse include:

- **Think-pair-share.** This approach can be used in a variety of instructional circumstances to encourage students to engage in mathematics independently and then share their results with a partner.
- **Numbered heads.** When students are working in groups of three or four, each can be assigned a number. Students know that any member of the group may be called on to provide a response, so everyone must have the same level of understanding. When teachers lead classroom discourse, they can use a spinner or another method to randomly select a group and group member (number) to respond.

Teachers must consider which classroom organizational structure (pairs, small-group, or whole-class) best serves the intended learning outcomes and classroom discourse objectives and which supports or hinders student engagement in classroom discussions.

When **students with diverse abilities** are grouped together, those with advanced abilities can support the learning of those with lower ability levels. However, in this arrangement, the lower-ability students may become listeners instead of contributors to the discussion.

When **students with similar abilities** are grouped together, regardless of level, those who may otherwise not have had an opportunity to talk mathematically can share and exchange ideas as they engage in mathematics. This arrangement can also free up teacher time to give extra attention to students who need it.

Teachers play a pivotal role in monitoring the performance of student groups by supporting and intervening as needed. To set students up for success, teachers can alert them that they will be asked to present an explanation to the class, then give them time to organize their thoughts and approaches with help from peers and, if needed, from the teacher. Teachers can encourage students to practice what they will say as they work with group members or with other students.

*Teachers play a **pivotal role** in monitoring the performance of student groups by **supporting and intervening** as needed.*

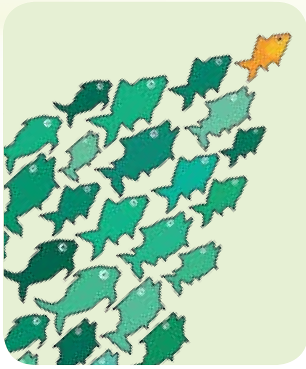
Strategy 7: Use a variety of pedagogical strategies to engage all students in whole-class, teacher-led mathematics discussion

To support mathematical discourse, teachers must always bear in mind which students volunteer most often, which students assume leadership roles in small and whole groups, and which students dominate the interactions.

Teachers must also avoid always calling on the students with the correct answers first. Students benefit from grappling with mathematical ideas, so points of confusion or misunderstanding help spur conversation and engage students in mathematical sense making as they question, negotiate, and rethink or confirm their ideas.

Teachers can use a variety of methods to gather information from the whole class or call on students.

- **Thumbs up/thumbs down.** Teachers pose a question or problem that has a dichotomous answer (yes/no, true/false, X or Y) and ask students to respond, using thumbs up to represent one choice and thumbs down to represent the other. This method gives every student an opportunity to respond and allows teachers to assess student understanding individually and collectively.
- **Response sticks.** Teachers write each student's name on a Popsicle stick or similar item, place the sticks in a container, and randomly select students by choosing a stick. The sticks can be removed from the container after use so that no student is selected more than once, or they can be returned to the container so that students can be called on repeatedly. This method reinforces the expectation that all students are expected to contribute to the learning environment.
- **Classroom response systems or other digital tools.** Teachers can use classroom response systems to gather immediate feedback from students. They can pose predetermined questions, typically in a true/false or multiple-choice format, and ask students to respond using a clicker, website, or text message. The results can be displayed as a graph or chart. This method allows teachers to gauge and compare student understanding and allows students to engage in conversation to clarify their understanding and reach consensus on the appropriate answer.



Planning and Leading Mathematical Discourse

To engage students in productive mathematical conversations, teachers must plan, initiate, and orchestrate discourse in ways that encourage student learning. It is not enough just to prepare the content of a math lesson. Teachers must select **worthwhile mathematical tasks** that provide opportunities for students to engage in rich mathematics discussions.

Smith and Stein (*2011*) identify five practices for orchestrating productive mathematical discourse:

- **Anticipating.** *"Actively envision how students might approach the mathematics task they will work on"(p. 8)*. For example, teachers can consider the following questions:
  - "How might students interpret the problem?"
  - "What different strategies might they use?"
  - "What specific aspects of the subject matter do I want them to understand?"
  - "What errors or misconceptions might they make?"
- **Monitoring.** *"[Pay] close attention to students' mathematical thinking and solution strategies as students work the task"(p. 9)* individually or in small groups. For example, as teachers walk around the room to observe and interact with students, they can use a tablet to note which students use expected or unexpected strategies and when they use them. This helps teachers keep track of which student or group produces solutions and which ideas to emphasize during the whole-class discussion.
- **Selecting.** *"Select particular students to share their work with the rest of the class to get specific mathematics into the open for examination" (p. 10)*. The selected students can be alerted in advance to give them time to gather and organize their thoughts.
- **Sequencing.** *"Make decisions regarding how to sequence the student presentation" (p. 10)*. The goal is to maximize the connections between and among ideas. For example, a teacher may first call on a student or group with incorrect thinking or an incorrect answer to highlight a common misconception before the class discusses the correct answer.
- **Connecting.** *"Help students draw connections between their solution and other students' solutions as well as the key mathematical ideas in the lesson" (p. 11)*. For example, they can ask the following questions:
  - "How are these two ideas similar?"
  - "How are they different?"
  - "How does this second idea build on or extend the idea we've just heard?"

These five practices build on each other to help teachers orchestrate mathematical discourse in meaningful ways. Although it is not possible to anticipate every strategy students might present, the five practices provide a way to capture, make sense of, and organize mathematical discourse in ways to maximize learning.

Conclusion

Facilitating student engagement in mathematical discourse begins with the decisions teachers make when they plan classroom instruction. The tasks they use, the ways in which they organize the classroom, and the behaviors they model communicate expectations for classroom norms, including the ways students are expected to engage in classroom discussions.

During class, teachers facilitate productive mathematics discussion by asking key questions to direct or redirect the nature of the conversation. By doing this, teachers motivate and encourage students and can strategically intervene to maximize student learning and mathematics language development.

Facilitating student engagement in mathematical discourse begins with the decisions teachers make when they plan classroom instruction.

References

Kersaint, G., Thompson, D.R., and Petkova, M. *Teaching Mathematics to English Language Learners* (2nd ed.). New York: Routledge, 2012.

National Council of Teacher of Mathematics (1991). *Professional Standards for Teaching Mathematics*. Reston, VA: National Council of Teachers of Mathematics.

National Council of Teachers of Mathematics (2000). *Principles and Standards for School Mathematics.* Reston, VA: National Council of Teachers of Mathematics.

National Council of Teachers of Mathematics (2007). *Mathematics Teaching Today: Improving Practice, Improving Student Learning.* Reston, VA: National Council of Teachers of Mathematics.

National Council of Teachers of Mathematics (2014). *Principles to Action: Ensuring Mathematical Success for All.* Reston, VA: National Council of Teachers of Mathematics.

Pirie, S.E.B. "Crossing the gulf between thought and symbol: Language as (slippery) stepping-stones." In Steinbring, H., Bartolini Bussi, M.G., and Sierpinska, A. (Eds.), *Language and Communication in the Mathematics Classroom* (pp. 7–29). Reston, VA: National Council of Teachers of Mathematics, 1998.

Smith, M.S. and Stein, M.K. *5 Practices for Orchestrating Productive Mathematics Discussions.* Reston, VA: National Council of Teachers of Mathematics, 2011.

Thompson, D.R., Kersaint, G., Richards, J.C., Hunsader, P.D., and Rubenstein, R.N. *Mathematical Literacy: Helping Students Make Meaning in the Middle Grades.* Portsmouth, NH: Heinemann, 2008.

Table 2. Questions to promote mathematical thinking and discourse

Help students work together to make sense of mathematics.	
What strategy did you use?	Would you ask the rest of the class that question?
Could you share your method with the class?	What part of what he said do you understand?
Would someone like to share ___?	Can you convince the rest of us that that makes sense?
What do others think about what Alexis said?	Can someone retell or restate Brandon's explanation?
Do you agree? Disagree?	Did you work together? In what way?
Would anyone like to add to this?	Have you discussed this with your group? With others?
Did anyone get a different answer?	Where would you go for help?
Did everybody get a fair chance to talk, to use the manipulatives, or to be recorded?	How could you help another student without telling the answer?
How would you explain ___ to someone who missed class today?	*Refer questions raised by students back to the class.*

Help students persevere.	
Have you tried making a guess?	What else have you tried?
Would another recording method work as well or better?	Is there another way to (draw, explain, say) that?
Give me another related problem. Is there an easier problem?	How would you explain what you know right now?

Help students learn to reason mathematically.	
How did you begin to think about this problem?	Can you explain this part more specifically?
What is another way you could solve this problem?	Does that always work?
How could you prove that?	Is that true for all cases?
Can you explain how your answer is different from or the same as Michael's?	How did you organize your information? Your thinking?
Let's see if we can break it down. What would the parts be?	

Help students evaluate their own processes, activities, and program.	
What do you need to do next?	What have you accomplished?
What are your strengths and weaknesses?	Was your group participation appropriate and helpful?

Help students with problem comprehension.	
What is this problem about? What can you tell me about it?	Would you please explain that in your own words?
Do you need to define or set limits for the problem?	What assumptions do you have to make?
How would you interpret that?	What do you know about this part?
Would you please reword that in simpler terms?	Which words were most important? Why?
Is there something that can be eliminated or that is missing?	

Help students learn to conjecture, invent, and solve problems.	
What would happen if ___? What if not?	Do you see a pattern?
What are some possibilities here?	Where could you find the information you need?
How would you check your steps or your answer?	What did not work?
How is your solution method the same as or different from Natalia's?	Other than retracing your steps, how can you determine if your answers are appropriate?
What decision do you think he or she should make?	How did you organize the information? Do you have a record?
How could you solve this using (tables, trees, lists, diagrams, etc.)?	What have you tried? What steps did you take?
How would it look if you used these materials?	How would you draw a diagram or make a sketch to solve the problem?
Is there another possible answer? If so, explain.	How would you research that?
Is there anything you've overlooked?	How did you think about the problem?
What was your estimate or prediction?	How confident are you in your answer?
What else would you like to know?	What do you think comes next?
Is the solution reasonable, considering the context?	Did you have a system? A strategy? A design? Explain it.

Help students learn to connect mathematics, its ideas, and its applications.	
What is the relationship of this to that?	Have we ever solved a problem like this before?
What uses of mathematics did you find in the newspaper last night?	What is the same? What is different?
Did you use skills or build on concepts that were not necessarily mathematical? What were they?	What ideas have we explored before that were useful in solving this problem?
Is there a pattern?	Can you write another problem related to this one?
Where else would this strategy be useful?	How does this relate to ___?
Is there a general rule?	Is there a real-life situation where this could be used?
How would your method work with other problems?	What other problem does this seem to lead to?

Help students rely more on themselves to determine whether something is mathematically correct.	
Is this a reasonable answer?	Does that make sense?
Why do you think that? Why is that true?	Can you draw a picture or make a model to show that?
How did you reach that conclusion?	Does anyone want to revise his or her answer?
How were you sure your answer was right?	

Help students focus on the mathematics from activities.	
What was one thing you learned (or two, or more)?	Where would this problem fit on our mathematics chart?
How many kinds of mathematics were used in this investigation?	What were the mathematical ideas in this problem?
What is mathematically different about these two situations?	What are the variables in this problem? What stays constant?

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